CMSC201
Computer Science I for Majors

Lecture 21 – Recursion (Continued)

Last Class We Covered

- Recursion
  - Recursion
    - Recursion
- Stacks
- Parts of a recursive function:
  - Base case: when to stop
  - Recursive case: when to go (again)

Any Questions from Last Time?

Today's Objectives

- To gain a more solid understanding of recursion
- To explore what goes on "behind the scenes"
- To examine individual examples of recursion
  - Binary Search
  - Fibonacci Sequence
- To better understand when it is best to use recursion, and when it is best to use iteration

Review of Recursion

What is Recursion?

- Solving a problem using recursion means the solution depends on solutions to smaller instances of the same problem

- In other words, to define a function or calculate a number by the repeated application of an algorithm

Recursive Procedures

- When creating a recursive procedure, there are a few things we want to keep in mind:
  - We need to break the problem into smaller pieces of itself
  - We need to define a “base case” to stop at
  - The smaller problems we break down into need to eventually reach the base case

"Cases" in Recursion

- A recursive function must have two things:

- At least one base case
  - When a result is returned (or the function ends)
  - "When to stop"
- At least one recursive case
  - When the function is called again with new inputs
  - "When to go (again)"

Code Tracing: Recursion

Stacks and Tracing

- Stacks will help us track what we are doing when tracing through recursive code
- Remember, stacks are **LIFO** data structures
  - Last In, First Out
- We'll be doing a recursive trace of the summation function

Summation Funcion

- The addition of a sequence of numbers
- The summation of a number is that number plus all of the numbers less than it (down to 0)
  - Summation of 5: 5 + 4 + 3 + 2 + 1
  - Summation of 6: 6 + 5 + 4 + 3 + 2 + 1
- What would a recursive implementation look like? What's the base case? Recursive case?

Summation Function

```
def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)
```

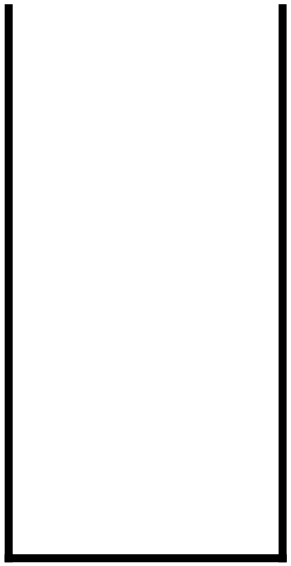
Base case:
Don't want to go below 0
Summation of 0 is 0

Recursive case:
Otherwise, summation is
num + summation(num-1)

```
main()

def main():
  summ(4)


def summ(num):
   if num == 0:
     return 0
   else:
     return num + summ(num-1)
```



```
main()

def main():
    summ(4)


def summ(num):
    if num == 0:
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```
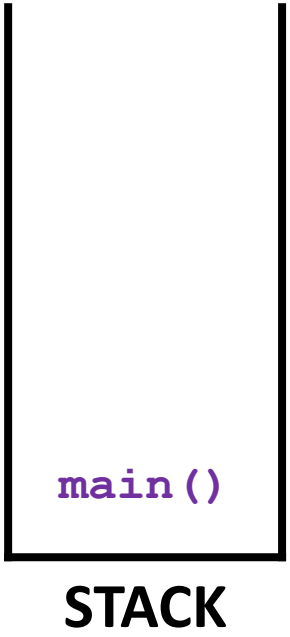
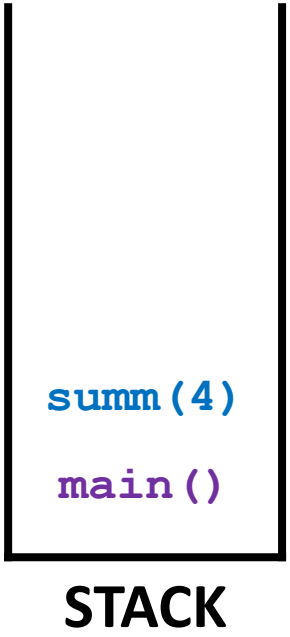



```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

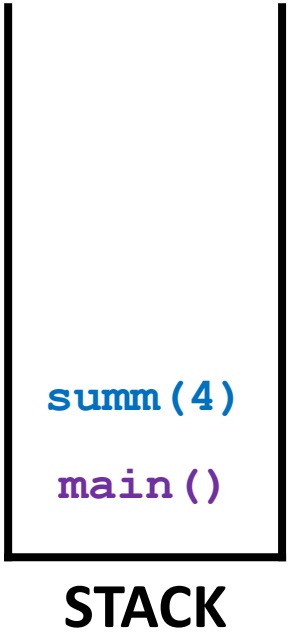
num: 4



```
main()

def main():
    summ(4)
                num = 4

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 4

This is a local variable. Each time the **summ()** function is called, the new instance gets its own unique local variables.



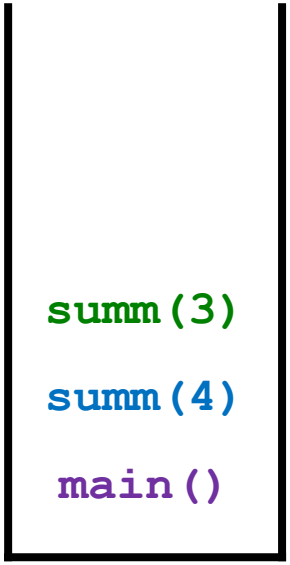
```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3



STACK

```
main()
```

```
def main():
    summ(4)
```

num = 4

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 4

num = 3

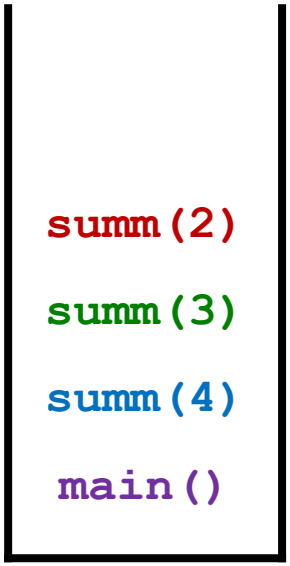
```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 3

num = 2

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 2



STACK

```
main()
```

```
def main():
    summ(4)
```

num = 4

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 4

num = 3

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 3

num = 2

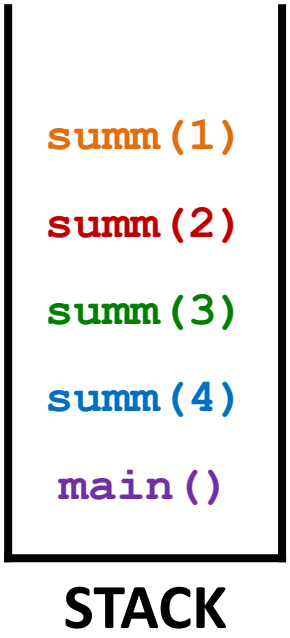
```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 2

num = 1

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num +
          summ(num-1)
```

num: 1



STACK

```
main()
```

```
def main():
    summ(4)
```

num = 4

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 4

num = 3

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 3

num = 2

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 2

num = 1

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num +
          summ(num-1)
```

num: 1

num = 0

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num +
          summ(num-1)
```

num: 0

summ(0)
summ(1)
summ(2)
summ(3)
summ(4)
main()

STACK

```
main()
```

```
def main():
    summ(4)
```

num = 4

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 4

num = 3

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 3

num = 2

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 2

num = 1

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num +
          summ(num-1)
```

num: 1

num = 0

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num +
          summ(num-1)
```

num: 0

summ(0)
summ(1)
summ(2)
summ(3)
summ(4)
main()

STACK

```
main()
```

```
def main():
    summ(4)
```

num = 4

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 4

num = 3

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 3

num = 2

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num: 2

num = 1

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num +
          summ(num-1)
```

num: 1

num = 0

return 0

```
def summ(num):
    if num == 0:
        return 0
    else:
        return num +
          summ(num-1)
```

num: 0

```
return 0
```

POP!

summ(1)

summ(2)

summ(3)

summ(4)

main()

STACK

```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3

num = 2

num: 2

num = 1

num: 1

return 1

POP!

POP!

summ(2)

summ(3)

summ(4)

main()

STACK

return 1 + 0 (= 1)

```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        return num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3

num = 2

num: 2

return 3

POP!

POP!

POP!

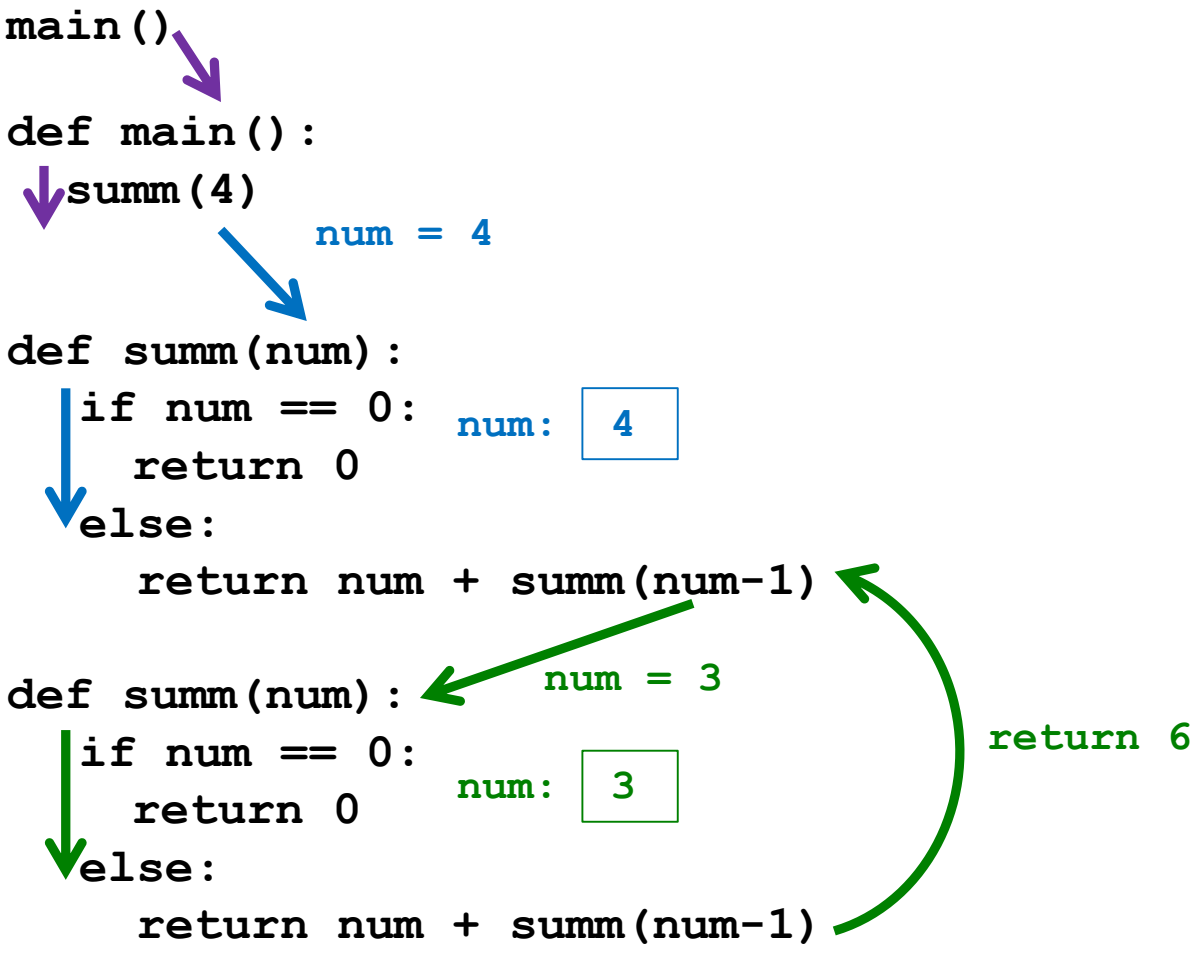
summ(3)

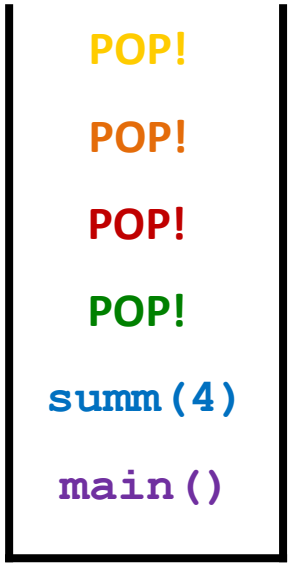
summ(4)

main()

STACK

return 2 + 1 (= 3)





STACK

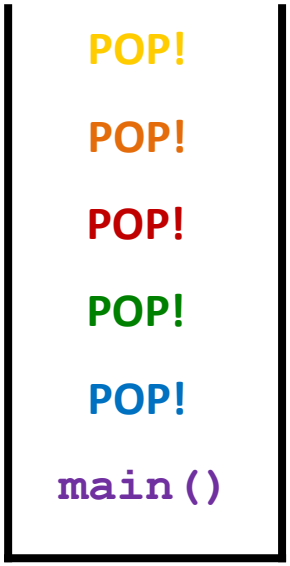
return 3 + 3 (= 6)

```
main()

def main():
  summ(4)

def summ(num):
  if num == 0:
    return 0
  else:
    return num + summ(num-1)
```

num = 4

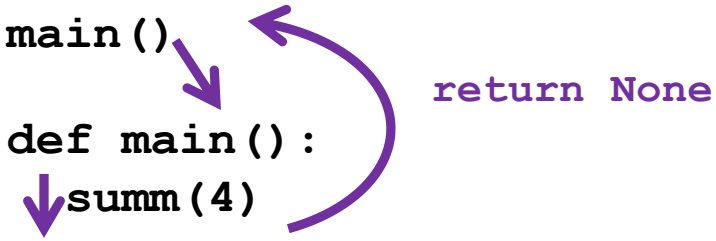
return 10

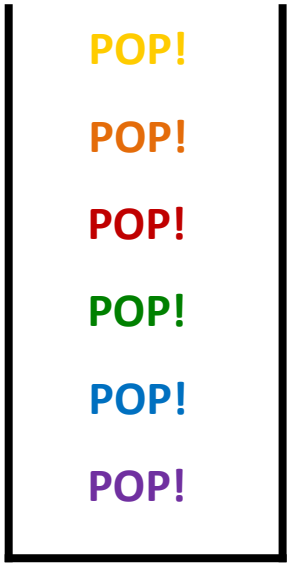
num: 4



STACK

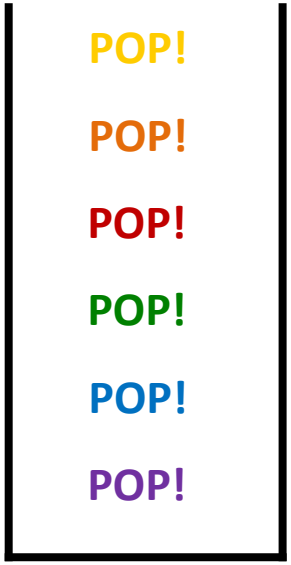
```
return 4 + 6 (=10)
```






```
return None
```

The stack is empty!




```
return control
```

Returning and Recursion

Returning Values

- If your goal is to return a final value
  - Every recursive call must return a value
  - You must be able to pass it "back up" to **`main()`**
  - In most cases, the base case should return as well
- Must pay attention to what happens at the "end" of a function.

```
main()

def main():
    summ(4)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)

def summ(num):
    if num == 0:
        return 0
    else:
        num + summ(num-1)
```

num = 4

num: 4

num = 3

num: 3

num = 2

num: 2

num = 1

num: 1

num = 0

num: 0

summ(0)
summ(1)
summ(2)
summ(3)
summ(4)
main()

STACK

Does this work? What's wrong?

Binary Search

Searching

- Given a list of sorted elements (e.g., words), find a specific word as quickly as possible

- We could start from the beginning and iterate through the list until we find it
  - But that could take a long time!

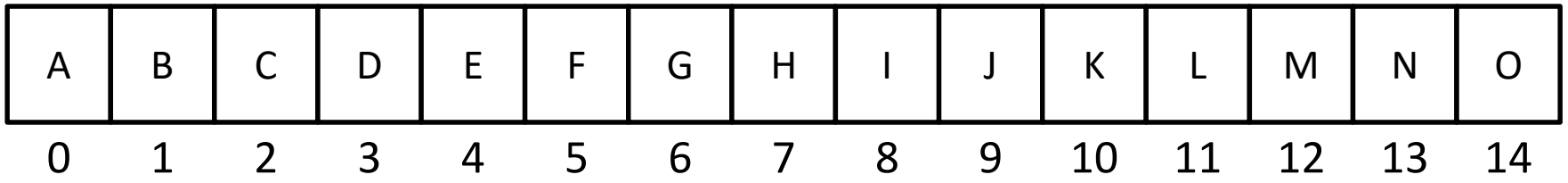
Binary Search

- Uses a “divide and conquer” approach

- Go to the middle, and compare the element there to the one we’re looking for
  - If it’s larger, we know it’s not in the last half
  - If it’s smaller, we know it’s not in the first half
  - If it’s the same, we found it!

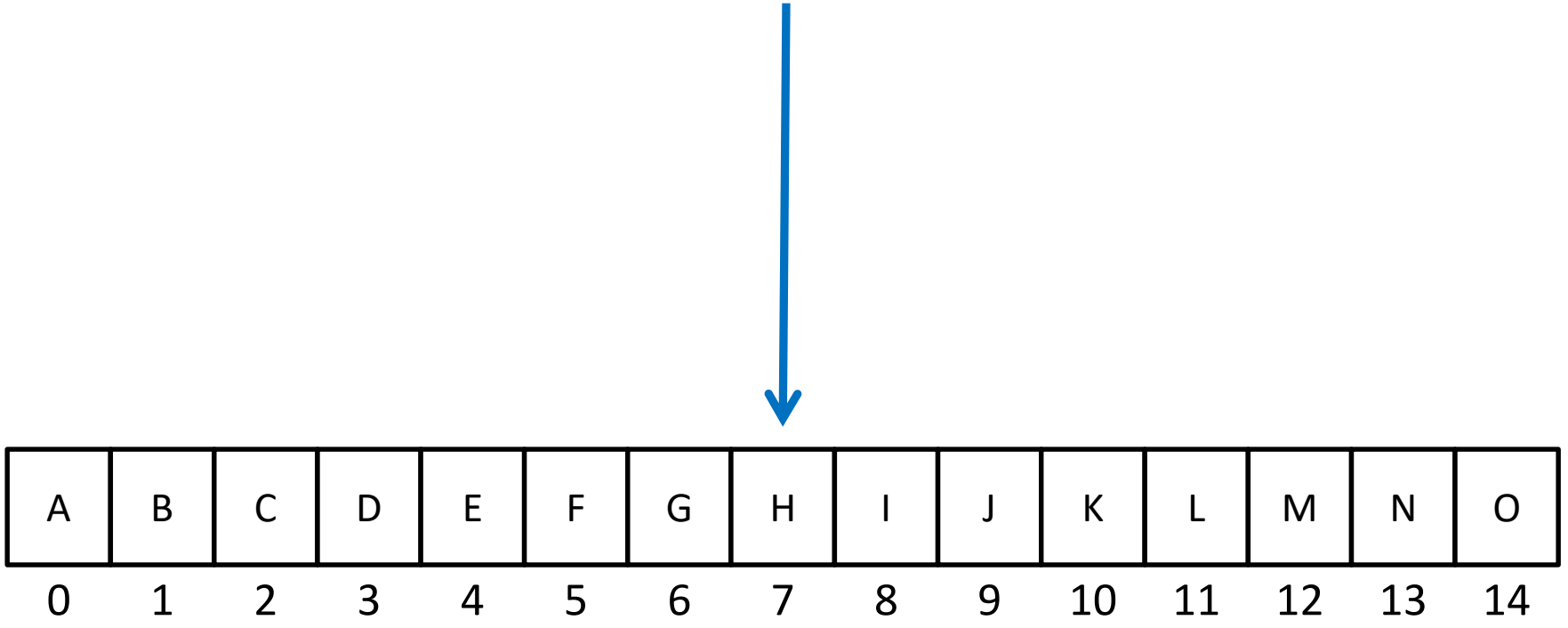
Binary Search Example

- Find the letter “J” using binary search



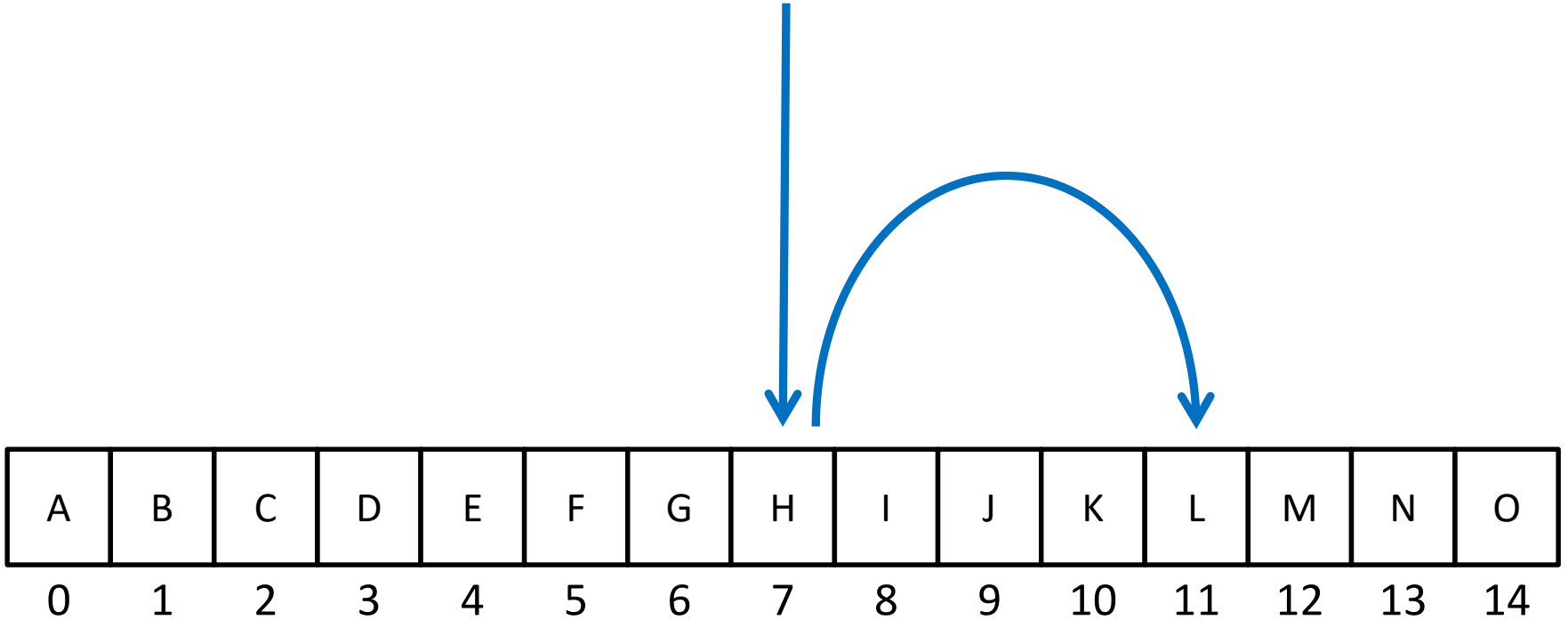
Binary Search Example

- Find the letter “J” using binary search



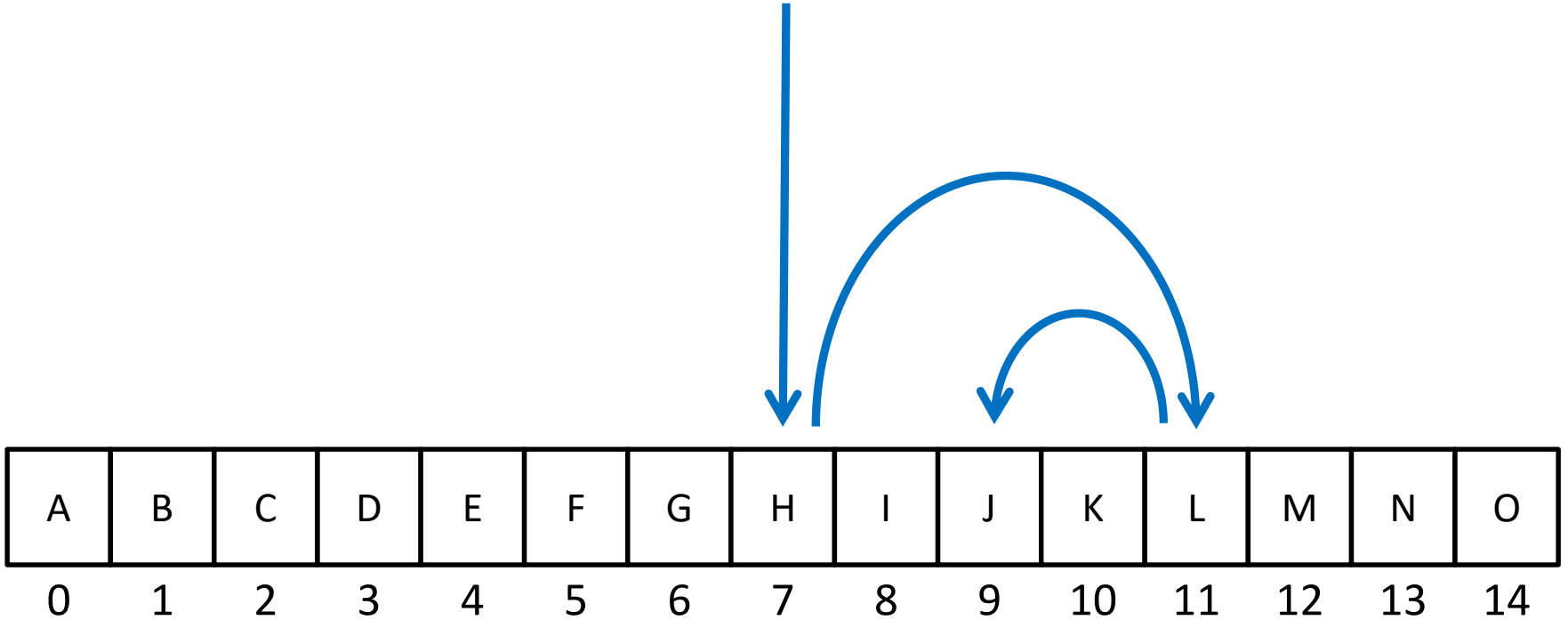
Binary Search Example

- Find the letter “J” using binary search



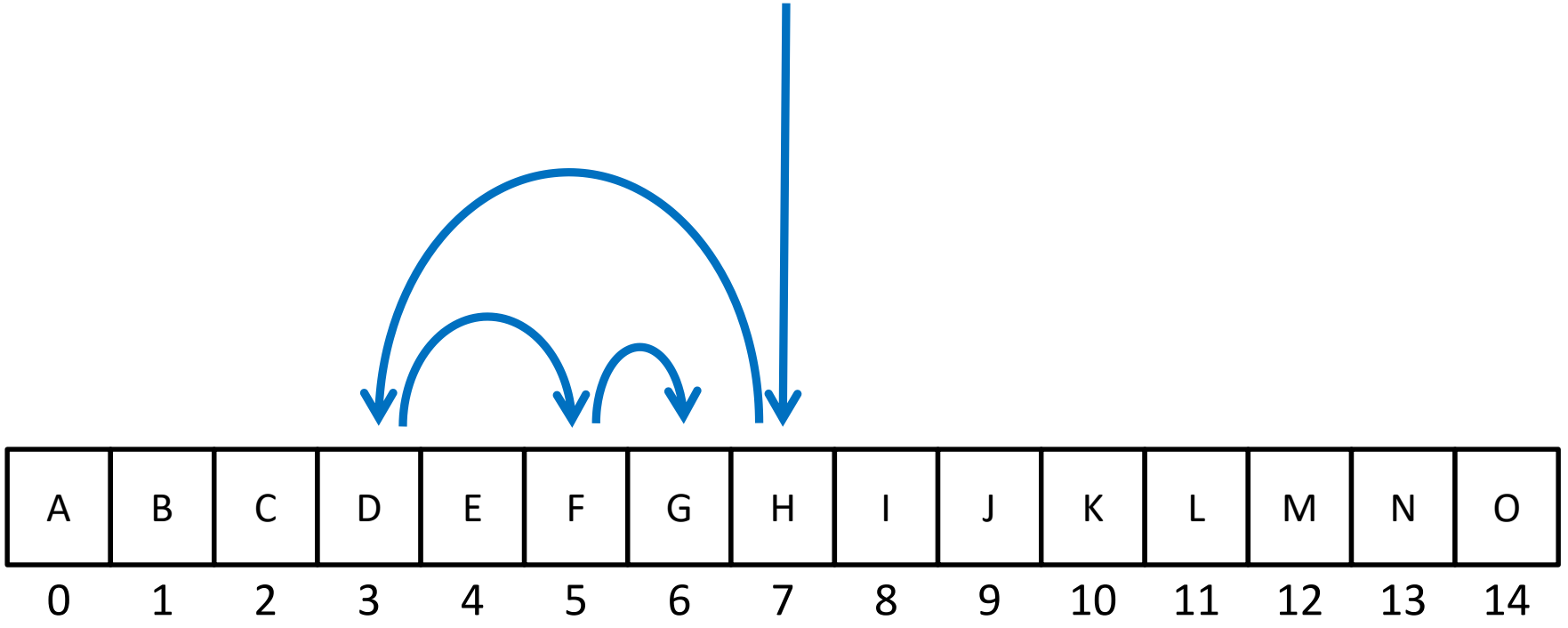
Binary Search Example

- Find the letter “J” using binary search



Binary Search Example

- Find the letter “G” using binary search



Binary Search

- Can be implemented using a **`while`** loop
  - But much more common to use recursion

- What is the base case?
- What is the recursive case?

Recursion vs Iteration

Recursion and Iteration

- Both are important
  - All modern programming languages support them
  - Some problems are easy using one and difficult using the other

- How do you decide which to use?

Use Iteration When…

- Speed and efficiency is an issue
  - Iteration doesn't push things onto the stack
- The problem is an obvious fit for iteration
  - Processing every element of a list (or 2D list)

Use Recursion When…

- Speed is not an issue
- The data being processed is recursive
  - A hierarchical data structure
- A recursive algorithm is obvious
- Clarity and simplicity of code is important

Fibonacci Sequences

Fibonacci Sequence

- Number series
- Starts with 0 or 1

- Next number is found by adding the previous two numbers together
- Pattern is repeated over and over (and over…)

Fibonacci Sequence

- Starts with 0, 1, 1
- Next number is …?

0 1 1 2 3 5 8 13 21 34 55

89 144 233 377 610 987 …

Time for…

LIVECODING!!!

Recursively Implement Fibonacci

- The formula for a number in the sequence:

```
fib(n) = fib(n-1) + fib(n-2)
```

- What is our base case?
- What is our recursive case?

Announcements

- Final is Thursday, December 15th (3:30 – 5:30)
  - If you have a conflict, fill out the Google Form
  - https://goo.gl/forms/We4cMotoNdbmCAoh2
- Homework 8 is out now
  - Last homework of the semester
  - Due this Wednesday – plan ahead!
- Project 2 will come out after Thanksgiving break